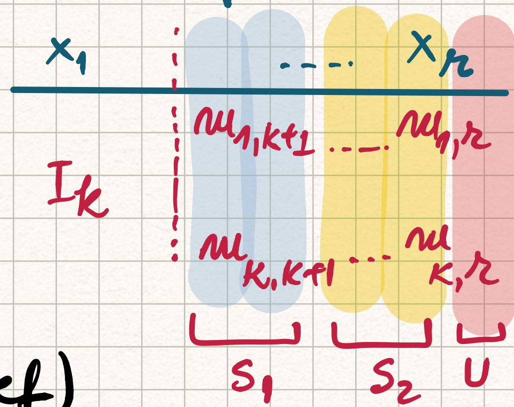
Building the Hori-Vafa mirror using the Przyalkowski trick.

Set-up: $\underset{\text{Fano}}{X} \subset \underset{\text{toric (GIT quot)}}{Y}$ a complete intersection



Convex partition w/ basis

↓

a partition of $[r]$ into $B, S_1 \dots S_p, U$

- $\{x_b \mid b \in B\}$ is a basis of $\mathbb{L}^\vee$
- ω is a non-negative linear comb. of $\{x_b\}$
- $S_i \neq 0$
- $\forall\, i = 1..p$, the line bundle $L_i = \sum_{j \in S_i} D_j$ is convex (nef)
- $\forall\, i = 1..p$, L_i is a non-negative linear comb. of $\{x_b \mid b \in B\}$.

$\Rightarrow$ The Givental / HV mirror to $X = \bigoplus_{i=1}^{p} L_i \subset Y$ is

$$f = x_1 + \dots + x_r - p \qquad \text{w/ constraints} \quad \begin{cases} \prod_{j=1}^{r} x_j^{m_{ij}} = 1 \quad \forall\, i = 1..k. & (1) \\ \sum_{j \in S_i} x_j = 1 \quad \forall\, i = 1..p. & (2) \end{cases}$$

The change of variables: 0. Choose 1 element $s_i \in S_i$ $\forall\, i = 1..p$.

1. $\forall\, i \in U$, introduce a variable y_i.
2. $\forall\, i = 1..p$ introduce a variable y_j for each $j \in S_i \setminus \{s_i\}$.
3. Set $y_{s_i} = 1$.
4. Solve (2) by setting $\begin{cases} x_j = \dfrac{y_j}{\sum_{l \in S_i} y_l} & \text{for } j \in S_i. \\ x_j = y_j & \text{for } j \in U. \end{cases}$
5. Express all other $x_\cdot$ in terms of $y_\cdot$ by solving (1).

Meta-statement

$\left\{ \begin{array}{c} \mathbb{Q}\text{-Fano 3folds} \\ X^n \end{array} \right\} \xleftrightarrow{1:1} \left\{ \begin{array}{c} \text{"Certain" Laurent} \\ \text{polynomials } f \\ \text{in } n \text{ variables} \end{array} \right\}$

deformation

mutation

Focus: $X \overset{CI}{\subset} Y_{toric}$

$\Updownarrow$

$\mathrm{Newt}(f) = P$ a Fano polytope

$\Updownarrow$

Can associate toric var.

$X \longleftrightarrow X_P$

$\mathbb{Q}$-Fano

deformation preserving anticon. ring.

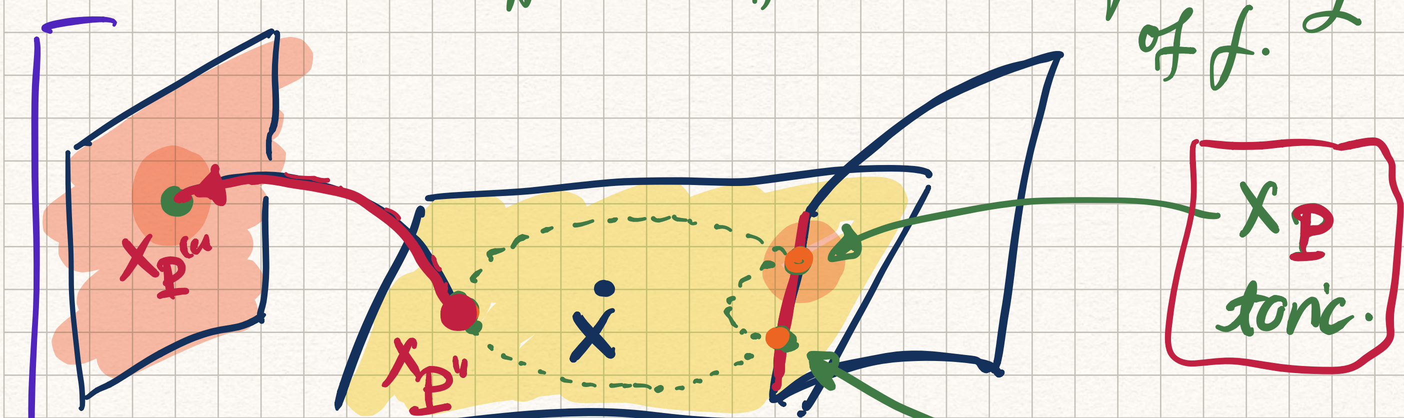
generally more singular.

One can attach periods to both sides (& the correspondence is stated in terms of these).

$$\pi_f(t) = \left(\frac{1}{2\pi i}\right)^n \int_{|x_1| = \dots = |x_n| = 1} \frac{1}{1 - t f(x_1 \dots x_n)} \frac{dx_1}{x_1} \dots \frac{dx_n}{x_n}$$

expand these periods int a power series

→ call the coefficients of t^m the period sequence of f.



$X_{\mathbb{P}'}$
toric.

everyone has the same Hilbert series

each individual component has a different Period sequence associated to it.

TODAY: Going " $\longrightarrow$ "

QFano $\longrightarrow$ Laurent polynomial.

Building the Givental / Hori-Vafa mirror to X using the Przyjalkowski trick.

Example:

$X_6 \subseteq \mathbb{P}(1123) \longrightarrow \dim X_6 = 2$

$\underbrace{}_{x_0 x_1 y z}$

$\mathcal{O}(6)$

The Hori-Vafa mirror to X_6 is

$f = \boxed{-1} + x_0 + x_1 + y + z$. w/ conditions

$$\begin{cases} x_0 \cdot x_1 \cdot y^2 \cdot z^3 = 1 & (1) \\ x_1 + y + z = 1 & (2) \end{cases}$$

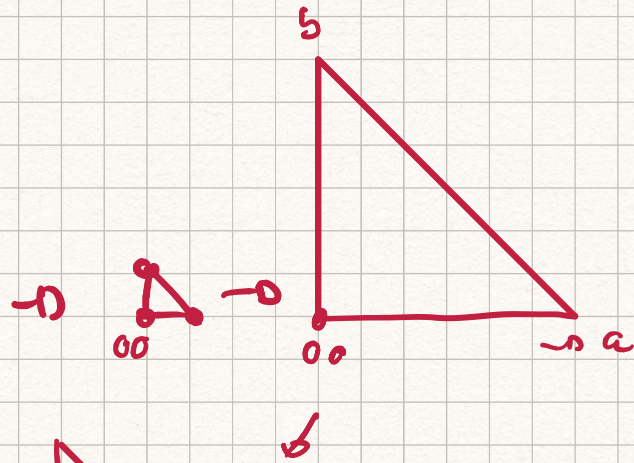
this is not a polyn. in 2 variables $\rightarrow$ I have to make a change of vars, to solve the system & obtain 2.

Change to a, b where

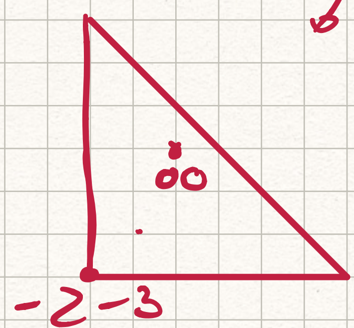
$$x_1 = \frac{1}{1+a+b}$$
$$y = \frac{a}{1+a+b} \qquad \text{\& substitute in (1).}$$
$$z = \frac{b}{1+a+b}$$

$$x_0 = \frac{1}{x_1 y^2 z^3} = \frac{(1+a+b)^6}{a^2 b^3}$$

$$\Rightarrow \boxed{f = x_0 = \frac{(1+a+b)^6}{a^2 b^3}}.$$



How to do this in general?



$$\begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline \to & 1 & 0 & 0 & 1 & -1 & 1 & 1 \\ \to & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ & & & S_1 & & S_2 & & U \end{array}$$

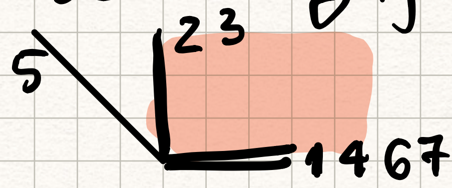
$\to$ the matrix of Y. $\quad$ dim $Y = 5$

$\to$ $w = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

$$L_1 = \begin{matrix} 0 \\ 1 \end{matrix} + \begin{matrix} 1 \\ 0 \end{matrix} = \begin{matrix} 1 \\ 1 \end{matrix}$$

$$L_2 = \begin{matrix} -1 \\ 1 \end{matrix} + \begin{matrix} 1 \\ 0 \end{matrix} = \begin{matrix} 0 \\ 1 \end{matrix}$$

$\to$ secondary fan



Find mirror for $X \in H^0(L_1 \oplus L_2) \subset Y$.

$L_1 \cap L_2$

$$f = x_1 + \dots + x_7 - 2$$

$$\left\{\begin{array}{l} x_1^{1}\, x_2^{0}\, x_3^{0}\, x_4\, x_5^{-1}\, x_6\, x_7 = 1 \\ x_2 x_3 x_5 = 1 \\ x_3 + x_4 = 1 \\ x_5 + x_6 = 1 \end{array}\right. \quad (1)$$

0. Choose $s_1 = 3$, $s_2 = 5$
1. y_7
2. y_4, y_6.
3. Set $y_3 = y_5 = 1$
4. For δ_1: $x_3 = \dfrac{y_3}{y_3 + y_4} = \dfrac{1}{1 + y_4}$

   $x_4 = \dfrac{y_4}{y_3 + y_4} = \dfrac{y_4}{1 + y_4}$

   δ_2: $x_5 = \dfrac{1}{1 + y_6}$

   $x_6 = \dfrac{y_6}{1 + y_6}$.

Substitute $\to$ in (1) & get.

$$f = \frac{1 + y_4}{y_6 y_4 y_7} + (1 + y_4)(1 + y_6) + y_7.$$

LP in 3 variables.