ERRATA FROM LAST TIME: f and $f+c$ give the same ~~period seq.~~ mirror.

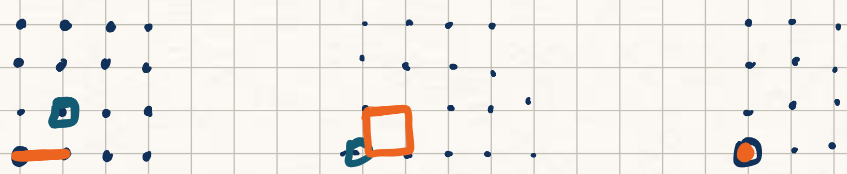
Yesterday:

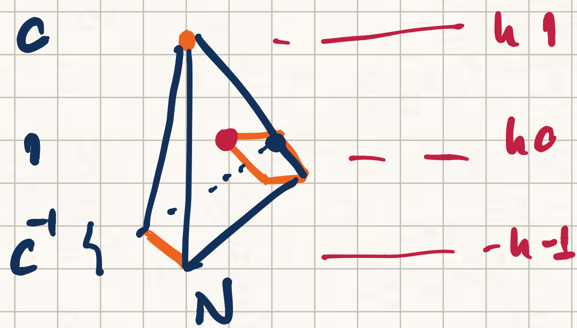
a partition of columns of Y

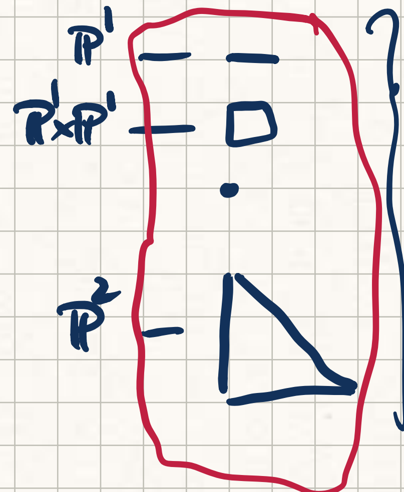
$$\begin{array}{c|cc|ccccc|c|ccc} & B & & S_i & & & & & U & & & \\ \| & 1&0&0&1&-1&1&1& \vdots & 1&0 \\ \| & 0&1&1&0&1&0&0& \vdots & 1&1 \end{array}$$

builds LP: $f = \underbrace{\frac{1+a}{abc}} + \underbrace{(1+a)(1+b)} + \underbrace{c}.$ $\to$ $U = \emptyset$

at height -1 at height 0 at height 1







What are these?

MOMENT POLYTOPES.

we need to consider an auxiliary object whose character lattice is N!

its fan lives in M.

How to reverse this:

1. Start w/ LP
2. Split LP into appropriate summands
3. Build weight matrix & line bdles.

this is Laurent inversion (Coates, Kasprzyk, Prince)

Write $X_P \xrightarrow{T_{C_i}} Y.$

Setup: • P a Fano polytope $\subset N$. $M = \mathrm{Hom}(N, \mathbb{Z})$

• A smooth projective toric variety Z given by a fan in M.

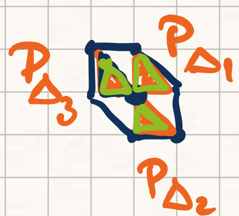
A SCAFFOLDING S of P is a set of torus-inv. nef divisors on Z, $S = \{\Delta_i \mid i \in I\}$ such that

$$P = \mathrm{Conv}(P_{\Delta_i} \mid \Delta_i \in S).$$

Z = shape, D_i = struts.

Example:

$$f = x + y + \frac{1}{x} + \frac{1}{y} + \frac{x}{y} + \frac{y}{x}$$



Choose $Z = \mathbb{P}^2$ $\parallel$

dP_6 $\mathbin{/\!/}$

this is a scaff. of X_P w/ shape $\mathbb{P}^2$.

How to build the weight matrix of Y?

Denote $R := \#\,\mathrm{rays}(\Sigma_Z)$

$\underline{k := |S|}$, $k + R = r$.

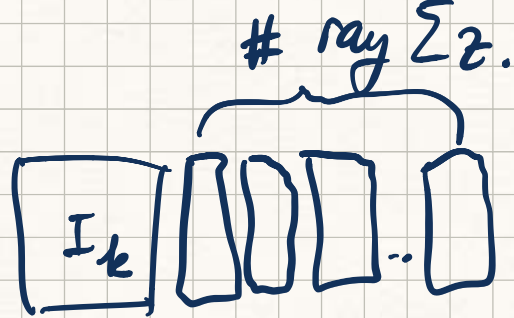
We build: a $k \times r$ - matrix as follows: M_{ij}.

• first an identity block.

$1 \le j \le k$, $m_{ij} = \delta_{ij}$

• (the s_i) $1 \le j \le R$, $m_{i,k+j}$ is det. by

$$\Delta_i = \sum_{j=1}^{R} m_{i,k+j} D_j$$



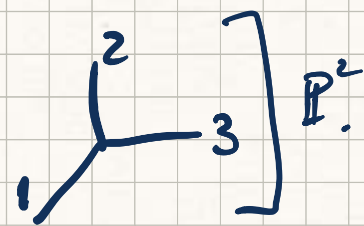
Then $w = (\underbrace{1\,1 \dots 1}_{k \text{ entries}})$

How many S_i? as many as $\rho(Z)$

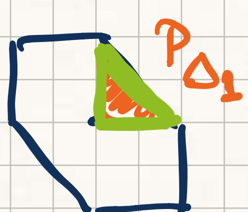
→ in the ex: 3×6 matrix

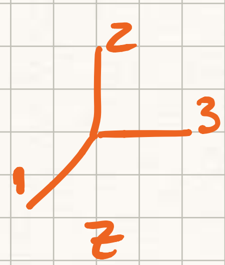
$$Y \quad \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \quad \begin{array}{c} L \\ 1 \\ 1 \\ 1 \end{array}$$

$\Delta_1 = \_\_\, D_1 + \_\_\, D_2 + \_\_\, D_3$



We have embedded
dP_6 into a 3fold Y as a hypersurface.
$\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$

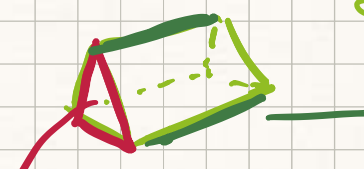




$\Delta_1 = \underline{1} \cdot D_1 + \underline{0} \cdot D_2 + \underline{0} \cdot D_3$

The last part of this lecture is about applying this construction to a specific polytope (see snapshots below)

Want something to cover

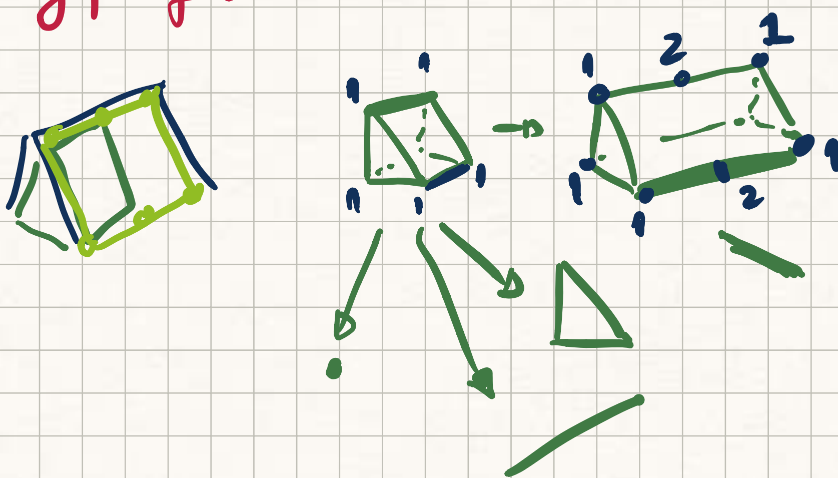


in one go.

polytope of $\mathbb{P}^2$

$\longrightarrow$ polytope of $\mathbb{P}^1$

$\Rightarrow$ the shape Z we are looking for is $\underline{\mathbb{P}^2 \times \mathbb{P}^1}$



what I obtain as a weight matrix is:

$$\begin{array}{ccc|cc|ccc} & & & \mathbb{P}^1 & & \mathbb{P}^2 & & \\ 1 & 0 & 0 & 1 & 1 & -3 & 2 & 2 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{array}$$

$\longrightarrow$ this is the matrix in the ex. session.

The line bundles:

$$L_1 = \begin{pmatrix}2\\2\\0\end{pmatrix} \qquad L_2 = \begin{pmatrix}1\\1\\1\end{pmatrix} \qquad w = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

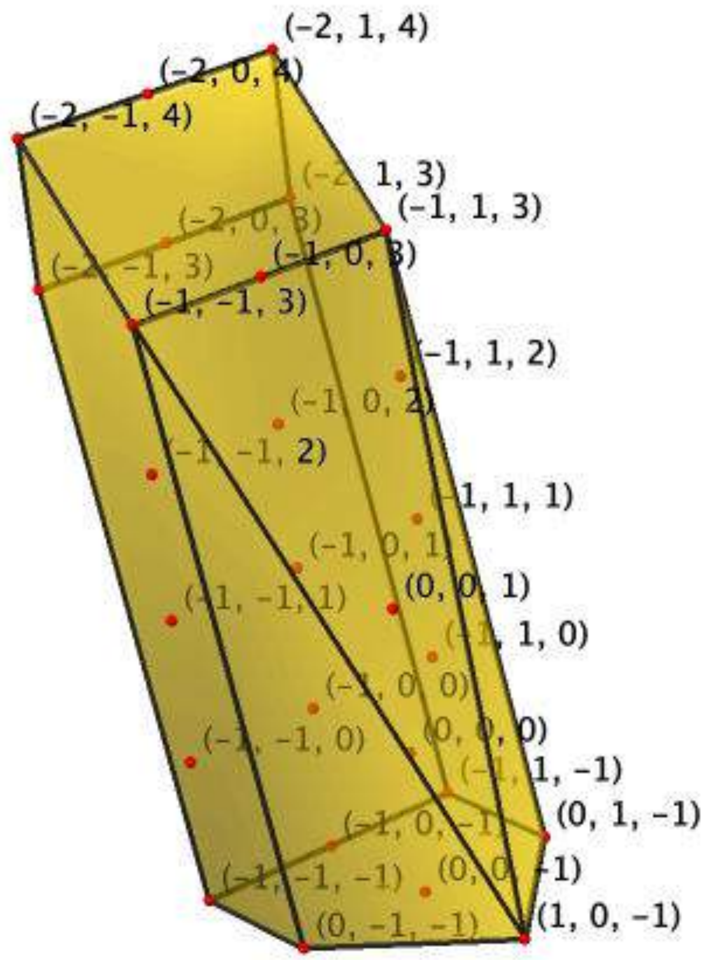
$\longrightarrow$ this has terminal singularities.

using -mk this embeds into wps as a codim 13 object.

I build $X = L_1 \cap L_2 \subset Y$. There is an adjunction formula:

$$-K_X = -K_Y - "L_1 - L_2" = \begin{matrix}4\\4\\2\end{matrix} - \begin{matrix}2\\2\\0\end{matrix} - \begin{matrix}1\\1\\1\end{matrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

– so in order to make sure I obtain a Fano variety, I choose $w = \begin{pmatrix}1\\1\\1\end{pmatrix}$.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